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Managing Urban Growth with Urban Growth Boundaries: A Theoretical Analysis*

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1. INTRODUCTION

According to Nelson and Duncan [20], the first urban growth boundary (UGB), or urban service area boundary, in the United States was drawn around Lexington, Kentucky, in 1958. Since then, the popularity of UGBs has grown rapidly. In 1973, the state of Oregon passed legislation requiring all cities to include UGBs in their comprehensive land use plans [14]. Similar requirements were passed in Washington State in 1989 (WAC 365-195-335). In 1997, the American Planning Association [1] prepared model state statutes for planning and zoning reform. As a part of these model statutes, the APA recommends that UGBs be established "to promote compact and contiguous development patterns that can be efficiently served by public services and to preserve or protect open space, agricultural land, and environmentally sensitive areas" [1, pp. 6–72]. Further, according to the APA, such urban growth areas "shall include or permit existing or proposed land uses at densities and intensities sufficient

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to permit urban growth that is projected for the region for the succeeding 20-year period'' [1, pp. 6–61].

Despite their increasing popularity, UGBs have received little formal analysis. In the introduction to their review of empirical research Knaap and Nelson [14] describe several ways in which UGBs could affect land values in Oregon but provide no formal analysis. Cho [5] developed a model in which UGBs cause congestion externalities and flatten urban rent gradients. He does not, however, address how a UGB should be drawn or expanded. Lee and Fujita [15] derive the conditions for the efficient configuration of an urban greenbelt. Their greenbelt, however, is characterized as a multifunctional park which provides urban amenities but does not constrain urban growth. A number of empirical studies have examined the effects of UGBs and similar policy instruments on land values without formal theoretical analysis; these are reviewed in Fischel [9].

In this paper we develop a theoretical model in which establishing a UGB and expanding it at distinct points in time can increase social welfare when a congestible public good is priced at average cost and urban infrastructure is fixed or lumpy. The model is cast in the framework of a monocentric, linear city with smoothly expanding bid rent functions that fall with distance from the urban center. As such, the model ignores many salient features of the urban landscape that planners must consider when designing actual UGBs. These include the supply and location of exceptionally productive farmland, forests, and other natural resources; the location and network structure of water, roads, and other forms of urban infrastructure; and the political geography of urban governments. Instead the model is based on the need to provide an abstract public good which rises in cost as the urban area grows. Further, the model assumes that the public good is produced with urban infrastructure that is fixed, or can only be augmented through lumpy investments, and that the public good is priced at average cost. Under these conditions the urban area is excessively large, as long as marginal cost exceeds average cost, and some form of growth control is warranted. Finally, the model assumes that UGBs represent an upper limit on the size, and thus the population, of an urban area and can only be adjusted at discrete points in time. This is based on the presumption that UGBs are elements of comprehensive plans that are costly to prepare and thus cannot be continuously revised. Based on these assumptions we show that (1) social welfare can be increased by setting an urban growth boundary; (2) an optimal boundary increases with infrastructure capacity and the planning horizon but decreases with development costs and agricultural rents; and (3) expansions in the boundary should be coordinated with investments in urban infrastructure.

Our paper builds on three previous papers, none of which specifically addresses the topic of UGBs as defined here. Brueckner [3] developed a model in which restrictions on the expansion of the urban boundary could increase land values and social welfare in the presence of a population externality. In Brueckner's model, however, the optimum urban boundary expands continuously and is not established *a priori*, as required by state statutes and recommended by the APA. In a more recent paper, Brueckner [4] developed a model that addresses the influence of impact fees as a method of infrastructure finance. In this model, infrastructure represents a nonrecurring, long-lived capital investment used to produce a constant level of public service. Under these assumptions, Brueckner shows that impact fees set at marginal cost generate an efficient urban growth path and that a switch to an impact-fee method of finance can result in discontinuous jumps or interruptions in the urban growth rate. Oum and Zhang [18] developed a model that examines the relationship between congestion tolls and capacity expansion costs for congestible transportation facilities with lumpy investments. Though not focused on urban growth, their model shares with ours a concern for managing congested facilities within lump-sum investment cycles.

We proceed as follows. First we develop a dynamic urban model with a congestible public good and a fixed stock of urban infrastructure and show that the optimum rate of growth depends on the marginal cost of providing the public good. Next we develop a model in which the public good is priced at average cost and show that a UGB can increase social welfare within a given planning horizon. Finally we characterize a model with an infinite horizon, lumpy infrastructure investments, and urban growth boundaries. With this model we show that UGBs can increase social welfare by managing growth within infrastructure investment cycles.

2. URBAN GROWTH WITH CONGESTIBLE PUBLIC GOODS

We begin model development by introducing a congestible public good; that is, a good that must be provided to all residents of the urban area but which is rival in consumption. Education, water and wastewater services, and police and fire protection are examples of these kinds of goods. Further we assume that local governments must maintain consumption of this good at a constant level, even though the cost of providing services may rise with population. This may be due to regulations imposed by a higher level of government or by municipal policy.[1]

Formally, we assume that the public good, z, is produced with a fixed stock of urban infrastructure, $\bar{K}$, and variable inputs, m, such that the

[1] In Florida, for example, local governments are required to adopt specific "level of service standards" (see, e.g., FAC 9J-5 and 9J-24). See also White [24].

public good production function can be specified as,

$$z = F(\bar{K}, m, n), \tag{1}$$

where n represents the urban population and $F_m > 0$ and $F_n < 0$, implying that the level of the public good rises with variable inputs and decreases with population.

Assuming that input prices are constant, and that local governments adjust the variable input to maintain z fixed at a constant level, a public good variable cost function can be specified as follows:

$$m = C(\bar{K}, n), \tag{2}$$

where m represents the level of the variable input necessary to maintain the level of the public good constant at $\bar{z}$ [which is omitted in (2)], and $C_n > 0$, which suggests that the cost of maintaining a constant level of the public good rises with population. This formulation of the cost function is similar to that used by Brueckner [4] and in the literature on clubs. Brueckner, however, ignores variable costs and focuses on capital cost; we assume capital costs are financed through lump-sum assessments and focus on variable costs. In addition, we assume $C_{nK} < 0$, which implies that marginal cost falls with the level of infrastructure.

Following Brueckner [4], we cast our model in the context of a linear open city of unit width with perfectly competitive factor and product markets. In such a model, all consumers have identical utility functions, which have as arguments a numeraire good, c, land, l, and the public good, z. Residents located x miles from the CBD must allocate their income, $y(t)$, so as to pay for the numeraire good, land rents, r, and transportation costs, kx, where k represents commuting cost per mile, and t represents time. If all residents consume one unit of land, the price of the numeraire good is one, and public good costs are paid by land owners, then, using the budget constraint, a representative utility function can be written as

$$v(c, l, z) = v(y(t) - r - kx, l, \bar{z}) = u(t), \tag{3}$$

where under open city assumptions, resident utility must equal exogenous utility levels $u(t)$. Under these conditions, bid rents can be expressed as

$$r = r(t, x), \tag{4}$$

where $r_x < 0$, and $r_t > 0$ as long as $y_t > (u_t/v_c)$, which implies that the growth rate of income is higher than the growth rate of utility divided by the marginal utility of income.[2]

Optimal Urban Growth

The objective of municipal governments is to maximize social welfare defined as the present value of aggregate land rents. Because the city is

[2] Differentiating (3) with respect to t yields $u_t = v_c(y_t - r_t)$. Thus $r_t = y_t - (u_t/v_c)$.

linear, is one land unit wide, and each resident consumes one unit of land, the distance to the urban boundary $\hat{x}$ at time t is equal to the urban population, $n(t)$. If B represents the distance from the CBD, located at one end of the city, to the opposite end of the municipal jurisdiction, then land between $\hat{x}$ and B remains in agricultural use. At time t, the urban population equals $n(t)$; at time $t + \Delta t$, the population becomes $n(t + \Delta t) = n(t) + n_t \Delta t$. Development cost over the period from t to $t + \Delta t$ thus equals $D n_t \Delta t$, where D represents the unit cost of converting land to urban use. If landowners incur the cost of providing the public good, the present value of aggregate land rents in the municipality can be expressed as

$$V = \int_0^T \left[\int_0^n r(t, x)\, dx + (B - n(t)) \cdot r^a - C\big(n(t), \bar{K}\big) - n_t D \right] e^{-it}\, dt, \tag{5}$$

where i denotes the discount rate and r^a represents non-urban land (agricultural) rent. In this formulation, social welfare is considered within the planning horizon $[0, T]$, where terminal time, T, could be infinite. The first term in the RHS of (5) represents the aggregate value of urban land rents, the second term represents the aggregate value of agricultural rents, the third term represents the variable cost of providing the public good, and the last term represents aggregate development costs. Each term is expressed in present value.

Since, by assumption, the level of the public good is fixed, the only choice variable is n, the urban population. After suppressing its time argument, the first order condition for n can be written as[3]

$$r(t, n) - C_n(\bar{K}, n) = r^a + iD. \tag{6}$$

Equation (6) states that to maximize aggregate land value, urban growth should proceed until urban rents minus the marginal cost of the public good equals agricultural rents plus the annual interest cost of development.

Urban Growth Under Average Cost Pricing

Equation (6) yields an implicit optimal urban growth trajectory: $n^* = n^*(t, \bar{K})$, which obtains when local governments do not impose growth controls and price the public good at marginal cost. Even if capital costs are paid by lump sum assessments or via an optimal impact fee, however,

[3] Equation (6) can be derived using calculus of variations. To do so, we define $G[\int_0^n r(t, x)\, dx + (B - n(t)) r^a - C(n(t), \bar{K}) - n_t D] e^{-it}$. Thus, $G_{n_t n_t} = 0$, $G_{n_t n} = 0$, $G_n = (r(t, n) - C_n(n, \bar{K}) - r^a) e^{-it}$, and $G_{n_t t} = iDe^{-it}$. Substituting these terms into the Euler condition $G_n = G_{n_t t} + G_{n_t n} n_t + G_{n_t n_t} n_{tt}$ yields Eq. (6). Since $G_{n_t n_t} = 0$, the second order condition, $G_{n_t n_t} \leq 0$, also holds.

such a pricing strategy will not generally yield a balanced budget. When average cost exceeds marginal cost, marginal cost pricing will yield a budget deficit; when marginal cost exceeds average costs, marginal cost pricing will yield a surplus. For these and other reasons, local governments typically price public goods at average cost. When local governments price public goods at average costs, urban growth occurs until $r(t, n) - [C(\bar{K}, n)/n] = r^a + iD$. Solving the above equation yields the equilibrium population growth path: $n^e = n^e(t, \bar{K})$. This equilibrium path is different from the optimal path $n^* = n^*(t, \bar{K})$ because the average cost $[C/n(t)]$ appears in place of marginal cost C_n. In the case where $C/n(t)$ is U-shaped, the equilibrium population can be larger or smaller than the optimal population depending on whether the rent function, which shifts up over time, intersects the marginal cost function to the left or to the right of the minimum point of the average cost function. When the marginal cost function lies below the average cost function, and services are priced at average cost, the urban population is inefficiently small, and development should be encouraged. When the average cost function lies below the marginal cost function, and public services are priced at average cost, the urban population is inefficiently large, and urban growth should be constrained.[4] Only under the latter conditions is it possible to increase social welfare by imposing a UGB. For the remainder of the paper, therefore, we assume that marginal cost always exceeds average cost, or that $C_{nn} > 0$ for all values of n.[5]

The context in which we conduct our analysis is illustrated in Fig. 1. In the figure, $r(t, n) - r^a + iD$ represents the net bid rent function at some arbitrary time; C_n and C/n represent the marginal and average public good cost functions, respectively. Because the public good is priced at average cost, the equilibrium size of the urban area at time t equals $n^e(t)$, whereas the efficient size of the urban area equals $n^*(t)$. The triangle *abc*, therefore, represents the efficiency loss at time t as a result of underpricing the public good.

3. URBAN GROWTH BOUNDARIES WITH FIXED URBAN INFRASTRUCTURE

We begin our analysis of UGBs with the assumption that the level of urban infrastructure is fixed, marginal cost exceeds average cost, public goods are priced at average cost, and thus the urban population is

[4] Once again, this result follows Brueckner [4].

[5] For ease of exposition, we assume that K is established at or before the time 0, when the first residents find it advantageous to locate in the city. Further, we presume that the marginal and average cost of the public good are equal and finite at $n = 0$. We thank an anonymous reviewer for this suggestion.

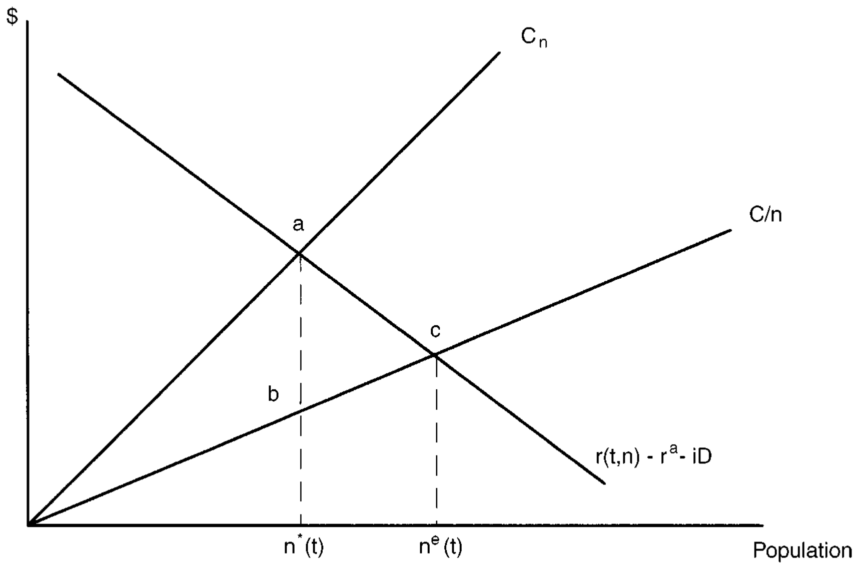

FIG. 1. The effects on social welfare of average cost pricing.

excessively large. The case for controlling urban growth based on these assumptions has been made before [8]. The standard policy recommendation is to adopt fees based on marginal cost or to slow (in a dynamic context) the rate of urban growth. In practice, however, both approaches are problematic. Although impact fees can be used to manage urban growth, they are more typically used to finance infrastructure and are often adopted with greater concern for equity than for efficiency [17, 19]. And, while building permits are sometimes rationed to slow the rate of urban growth, such rationing methods fail to address problems associated with lumpiness in public infrastructure (addressed in the subsequent section), complexities in transportation and utility networks, and difficulties in coordinating disparate public services. For these reasons, comprehensive land use plans are made at distinct points in time and revised periodically.[6] As a consequence, and by statute, UGBs have become integral elements of such discrete and periodic land use plans.

[6] Standard planning practice is described in Kaiser, Godschalk, and Chapin [13]. In a theoretical analysis of planning, Intriligator and Sheshinski [12] argue that the period between plan revisions depends on the complexity of the planning problem, the degree of uncertainty, and the costs and benefits of planning.

An Optimal Urban Growth Boundary

To develop a formal model of UGBs we begin by expressing the aggregate land value, and thus the social welfare function, as follows:

$$\begin{aligned} J = {} & \int_0^{t^b} \int_0^{n^e} r(t,x) e^{-it}\, dx\, dt - \int_0^{t^b} C(\overline{K}, n^e) e^{-it}\, dt \\ & + \int_{t^b}^{T} \int_0^{n^b} r(t,x) e^{-it}\, dx\, dt - \int_{t^b}^{T} C(\overline{K}, n^b) e^{-it}\, dt \\ & + \int_0^{t^b} (B - n^e) r^a e^{-it}\, dt + \int_{t^b}^{T} (B - n^b) r^a e^{-it}\, dt \\ & - \int_0^{t^b} n_t^e D e^{-it}\, dt, \end{aligned} \tag{7}$$

where n^b is the maximum population allowed by the UGB, t^b is the time at which the population level reaches the UGB population under average cost pricing, and T is some predetermined terminal date. Note that n^e is implicitly a function of time while n^b is constant.

The first and third terms on the RHS of (7) represent the present values of aggregate urban land rents before and after the UGB becomes binding. The second and fourth terms represent the present values of public service costs before and after the UGB. The fifth and sixth terms represent the present values of aggregate agricultural rents before and after the UGB. The last term represents the present value of development costs before the UGB. Development costs after the UGB becomes binding are zero.

By assumption, the path of urban growth follows n^e before the imposition of the boundary; thus $n^b = n^e(t^b)$, so we can write n^b as a function of t^b, and $n^b_{t^b} > 0$, which suggests that a UGB imposed at a later date will contain a larger population. Inverting the relationship yields $t^b = g(n^b)$, where $g_{n^b} > 0$. Substituting $t^b = g(n^b)$ into (7) yields

$$\begin{aligned} J = {} & \int_0^{g(n^b)} \int_0^{n^e} r(t,x) e^{-it}\, dx\, dt - \int_0^{g(n^b)} C(\overline{K}, n^e) e^{-it}\, dt \\ & + \int_{g(n^b)}^{T} \int_0^{n^b} r(t,x) e^{-it}\, dx\, dt - \int_{g(n^b)}^{T} C(\overline{K}, n^b) e^{-it}\, dt \\ & + \int_0^{g(n^b)} (B - n^e) r^a e^{-it}\, dt + \int_{g(n^b)}^{T} (B - n^b) r^a e^{-it}\, dt \\ & - \int_0^{g(n^b)} n_t^e D e^{-it}\, dt. \end{aligned} \tag{8}$$

Equation (8) includes a single decision variable, n^b, the population within the UGB.[7] The first order condition for the UGB is[8]

$$\int_{g(n^b)}^{T} \left(r(t, n^b) - C_n(\bar{K}, n^b) - r^a \right) e^{-it}\, dt = De^{-ig(n^b)}. \tag{9}$$

The above equation implies that the optimal UGB should be set at the population level, and thus the date, when the present value of urban rents, net of agricultural rents and the marginal cost of the public good—summed over the period between the imposition of the UGB and the terminal date—equals the present value of development cost.

The intuition behind Eq. (9) can be illustrated using Fig. 1 if, for expository purposes, T is set at infinity. When T is infinite, (9) becomes

$$\int_{g(n^b)}^{T} \left(r(t, n^b) - C_n(\bar{K}, n^b) - r^a - iD \right) e^{-it}\, dt = 0. \tag{9a}$$

The term in parentheses in (9a) represents, in Figure 1, the difference between the height of the rent curve and the height of the marginal cost curve at population n^b. This difference changes over time as the rent curve rises. To satisfy (9a), n^b must be chosen such that the difference in height, weighted by the discount factor, integrates to zero over the interval $[g(n^b), T]$. This means that the difference should start out negative and turn positive, or that the city is larger than optimal early in the interval and smaller than optimal late in the interval.[9]

The effect of the UGB on urban growth is illustrated in Fig. 2. In Fig. 2, the optimal growth path is illustrated by $n^*(t)$ and the equilibrium growth path by $n^e(t)$. The growth path, under average cost pricing and an optimal

[7] The same solution will result if the date at which the boundary is imposed, t^b, is chosen rather than n^b.

[8] Because the population path is continuous and differentiable over the planning horizon, until the terminal date, applying Leibniz's rule to take the derivative of (8) with respect to n^b causes all the integrals between $[0, g(n^b)]$ to disappear. For instance, the partial derivative of the first term is $g'(n^b) \int_0^{n^b} r(t, x) e^{-ig(n^b)}\, dx$ and the partial derivative of the third term is $-g'(n^b) \int_0^{n^b} r(t, x) e^{-ig(n^b)}\, dx + \int_{g(n^b)}^{T} r(t, x) e^{-it}\, dt$. Thus the parts of limited integral between $[0, g(n^b)]$ cancel out. This same canceling out occurs with the terms that contain r^a, B, C, and D. For the last term this effect leaves the expression $-De^{-ig(n^b)}$. The second order condition is

$$\int_{g(n^b)}^{T} (r_{n^b} - C_{n^b n^b}) e^{-it}\, dt$$
$$- \left(r(g(n^b), n^b) - C_{n^b}(\bar{K}, n^b) - r^a - iD \right) \cdot g_{n^b}(n^b) e^{-ig(n^b)} < 0.$$

[9] We thank Jan Brueckner for this explanation.

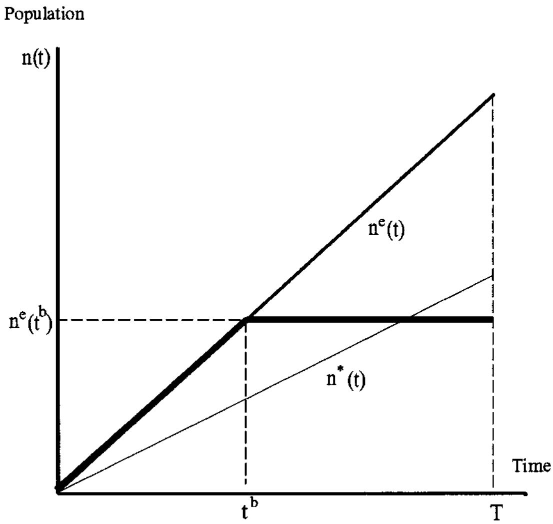

FIG. 2. Urban growth over a finite horizon with an optimal UGB.

UGB, follows $n^e(t)$ until time t^b, when growth stops at population level $n^e(t^b)$.

Properties of an Optimal UGB

Several properties of an optimal UGB can be derived by differentiating Eq. (9).[10] Using the second order condition, $J_{n^b n^b} < 0$, and holding other parameters constant reveals the following:

1. The optimal size of the UGB falls with development costs. That is,

$$\frac{dn^b}{dD} = \frac{e^{-ig(n^b)}}{j_{n^b n^b}} < 0. \tag{10}$$

[10] The total differential of (9) equals:

$$J_{n^b n^b}\, dn^b + \left(r(T, n^b) - C_{n^b}(\overline{K}, n^b) - r^a\right) e^{-iT}\, dT + \int_{g(n^b)}^{T} e^{-it}\, dt\, dr + \int_{g(n^b)}^{T} - e^{-it}\, dt\, dr^a$$

$$+ \int_{g(n)}^{T} \left(- C_{n^b K} e^{-it}\, dt\, d\overline{K} + \left(-e^{-ig(n^b)}\right) dD = 0\right.$$

2. The optimal size of the UGB falls with agricultural rents. That is,

$$\frac{\partial n^b}{\partial r^a} = \frac{\int_{g(n^b)}^{T} e^{-it}\, dt}{J_{n^b n^b}} < 0. \tag{11}$$

These two properties confirm expectations that the optimal size of an urban area should fall with development costs and agricultural rents—even when the urban area is constrained by a UGB.

3. The optimal size of the UGB increases with the stock of infrastructure if the marginal cost of providing the public good falls with the stock of infrastructure. That is,

$$\frac{\partial n^b}{\partial K} = \frac{\int_{g(n^b)}^{T} C_{nK} e^{-it}\, dt}{J_{n^b n^b}} > 0 \tag{12}$$

as long as $C_{nK} < 0$. This proposition has important policy implications. It suggests that UGBs should not be based exclusively on projected rates of urban growth but should be based in part on the projected absorption rate of infrastructure capacity. An urban area with considerable infrastructure capacity, for example, should establish a larger UGB than an urban area with more limited capacity, even if both urban areas are projected to grow at the same rate. It also implies that the UGB should be adjusted following an increase in infrastructure capacity.

4. The optimal size of the UGB increases with the terminal date of the planning horizon. That is,

$$\frac{\partial n^b}{\partial T} = \frac{-\left(r(T, n^b) - C_n(\bar{K}, n^b) - r^a\right)e^{-iT}}{J_{n^b n^b}} > 0. \tag{13}$$

Equation (13) states that the optimal UGB increases with the terminal date of the planning horizon as long as the expression in parentheses is positive, which requires that rents at the terminal date exceed the sum of the marginal cost of the public good and agricultural rents, evaluated at the UGB population. Intuitively, this mean that the size of the UGB increases with the terminal date as long as there are net gains in rents at the terminal date. This can be demonstrated as follows.

Using the Mean-Value Theorem, Eq. (9) can be expressed as

$$\left(r(\hat{t}, n^b) - C_n(\bar{K}, n^b) - r^a\right)e^{i\hat{t}} \cdot \left(T - g(n^b)\right) = De^{-ig(n^b)}, \tag{14}$$

where $g(n^b) < \hat{t} < T$. Thus, since $De^{-ig(n^b)} > 0$ and $T - g(n^b) > 0$, then $r(\hat{t}, n^b) > C_n(\bar{K}, n^b) + r^a$. This implies that at some date, $\hat{t}$, which occurs before the terminal date, rents must exceed the sum of marginal cost and agricultural rents. Further, since $r_t > 0$, rents at the terminal date must exceed rents at the date $\hat{t}$; that is, $r(T) > r(\hat{t})$. By transitivity, then, rents at the terminal date must exceed the sum of marginal cost and agricultural rents, and the partial derivative in (13) must be positive.

4. URBAN GROWTH BOUNDARIES WITH LUMPY INFRASTRUCTURE INVESTMENT

In the preceding model we showed how a UGB can increase social welfare by limiting the consumption of infrastructure capacity within a given time horizon. The model was structured such that without some form of growth control, too much land is developed because public services were priced at less than marginal cost. We then showed that it was preferable to prevent the development of land for which the present value of net rents, accumulated over the planning horizon, would be less than the present value of development costs. We did not demonstrate how such a horizon should be chosen. This issue can be addressed, however, by relaxing the assumption that the stock of infrastructure is permanently fixed and allowing the stock to increase through lump-sum investments. Since the UGB serves to ration infrastructure capacity, the period within which the UGB can serve as a rationing device is framed by the periodicity of infrastructure investment cycles. As demonstrated in Eq. (12), the size of the UGB should increase with the level of infrastructure capacity. If infrastructure capacity cannot be increased continuously, but can only be increased through lump-sum increments, then the dates of these lump-sum increments can serve as the dates that frame decisions regarding the UGB. That is, in a model with lumpy infrastructure, UGBs can serve as a rationing device within infrastructure investment cycles.

Critical to such a model is the presumption that urban infrastructure is best increased through lumpy investments. Problems associated with lumpy investments have been examined by economists for some time [2, 6, 10, 16, 22]. Apart from physical indivisibility, lumpy investments are optimal when transaction costs are high relative to operating costs or when capacity expansion exhibits economies of scale. Such features are common in urban transit systems [18, 23], wastewater pipes and treatment plants [11], and urban water facilities [21]. For these reasons, lumpiness in infrastructure is often the implicit logical foundation for growth management instruments such as impact fees, concurrency requirements, capital facilities planning, and urban growth boundaries [24].

In a formal model in which local governments simultaneously make decisions regarding investment in infrastructure and the size of UGBs,

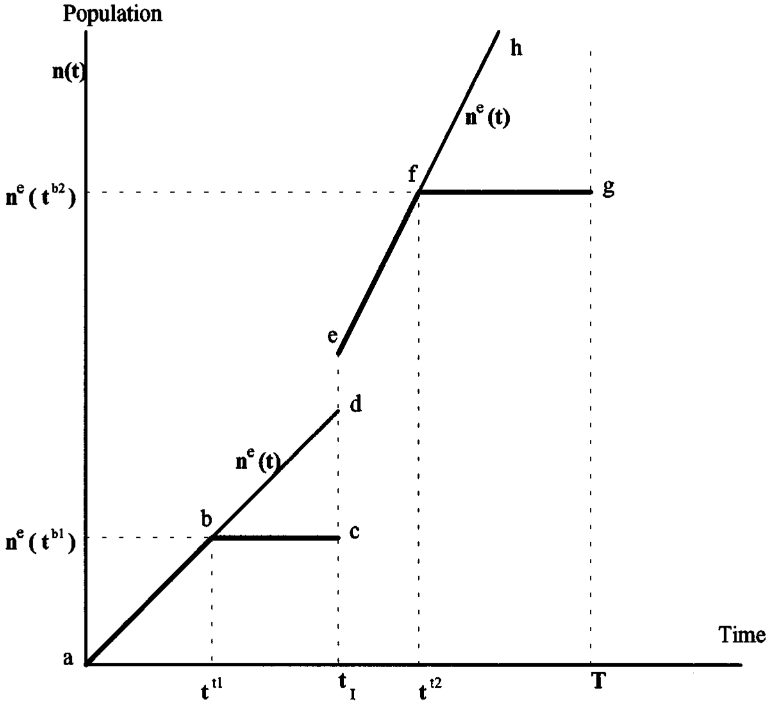

FIG. 3. Urban growth with infrastructure investment and a UGB.

standard comparative dynamic analysis is not fruitful.[11] The general implications of the model, however, can be illustrated using Fig. 3. As shown in the figure, the path urban growth follows if the public good is priced at average cost, without the imposition of UGBs, is adeh. With average cost pricing and optimal UGBs, urban growth follows the path abcefg. In this growth path, UGBs mitigate distortions due to average-cost pricing by interrupting growth at points b and f.

Figure 3 also illustrates that the optimal urban growth path requires the urban population to rise instantaneously following the investment in infrastructure. This stems from the need to utilize the abrupt increase in infrastructure capacity created by an infrastructure investment. Thus, the UGB should be released at the point of infrastructure investment and reset at a higher population level. As a result, the terminal points of the UGB planning horizons are determined by the length of the infrastructure cycle. This is a useful insight. Without justification, the APA [1] and Easley

[11] The details of such a model are available from the authors on request.

[7] suggest that UGBs should be based on a 10- to 20-year planning horizon. This model offers a framework for addressing this issue. Specifically, it suggests that the time of infrastructure investment, and thus the terminal planning horizon for the initial UGB, should be based on decisions regarding optimal dates of investment in infrastructure. Further it suggests that such dates occur when anticipated gains in urban rents exceed the anticipated marginal cost of investment, which includes both the capital cost of the investment and the change in the variable cost of the public service. When considering the planning horizon for a UGB, therefore, these factors should be considered.

The model outlined above, with the exception of the UGB, is similar to the model developed by Oum and Zhang [18], which examines traffic patterns within investment cycles in transportation infrastructure. Using numerical simulations, they show that traffic in an airport facility increases at a decreasing rate as congestion tolls rise, and that traffic increases abruptly following an investment in airport capacity. They also show that the extent to which marginal cost pricing yields revenues sufficient to cover the cost of capacity expansions depends critically on the initial level of capacity, the interest rate, the price elasticity of demand, and the rate of capacity congestion. Because we focus on variable cost and assume fixed costs are paid through a lump-sum assessment, we do not address these issues here; but it is likely that budgetary issues in an expanded version of our model would be affected by similar contextual parameters.

5. CONCLUSIONS

This paper has examined the problem of urban growth management with a congestible public good that must be maintained at some constant level. The examination was conducted by developing three increasingly complex models. In the first model, we showed that an efficient path of urban growth can be achieved by pricing the public good at marginal cost. This is a standard result. In the second model, we examined the problem of managing urban growth when the public good is priced at average cost. Here we showed that efficiency can be enhanced by imposing a UGB that balances efficiency losses that result from overpopulation before the UGB is binding against the underpopulation that results after the UGB restricts population growth. We also showed that the population contained within an optimal UGB increases with the extent of the planning horizon and the capacity of infrastructure. In our final model we examined the interrelationships between decisions regarding the UGB and investments in infrastructure. Here we showed that efficiency is enhanced by coordinating expansions in the UGB and investments in the stock of urban infrastructure. Such UGBs balance efficiency losses within infrastructure investment cycles and are released at the point of infrastructure investment. In this model, the path of urban growth is characterized by a cycle of continuous urban growth as infrastructure capacity is absorbed, zero urban growth when a UGB is binding, and rapid urban growth after infrastructure capacity is enhanced and the UGB is reset.

As we make clear through the incremental development of our models, the efficient imposition of UGBs depends critically on two assumptions: that investments in infrastructure are lumpy and that urban growth cannot be managed using marginal cost pricing (with lump-sum taxes or impact fees to cover capital costs) or by continuous and infinitely small extensions of the UGB. We do not prove here the validity of these assumptions or argue that UGBs are in fact imposed on the logic we derive from these assumptions. Instead, we observe that local governments impose discretely adjusted UGBs (sometimes in accordance with state laws), and we characterize conditions under which such UGBs can improve economic efficiency. In our model, UGBs serve as blunt instruments for rationing public service capacity when marginal cost pricing is not possible and a more refined instrument is unavailable. Whether in fact UGBs serve as such an instrument, and whether in fact UGBs increase social welfare, we leave for future research.

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